

Roll No _____ (To be filled in by the candidate)

(Academic Sessions 2019 – 2021 to 2021 – 2023)

MATHEMATICS

223-1st Annual-(INTER PART – II) Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – I

Maximum Marks : 20

PAPER CODE = 8191 LHR-12-1-23

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	The perimeter P of a square as a function of its area A is given as : (A) 4A (B) $4\sqrt{A}$ (C) 2A (D) $2\sqrt{A}$
2	Domain of cosine function $y = \cos x$ is : (A) Real numbers (B) $[-1, 1]$ (C) $(0, \infty)$ (D) $]-1, 1[$
3	If $y = \tanh^{-1} x$, then $\frac{dy}{dx} =$: (A) $\frac{1}{1+x^2}$ (B) $\frac{1}{1-x^2}$ (C) $\frac{-1}{1+x^2}$ (D) $\frac{-1}{1-x^2}$
4	$\frac{d}{dx}(a^{\lambda x}) =$: (A) $a^{\lambda x}$ (B) $a^{\lambda x} \ln a$ (C) $\lambda a^{\lambda x} \ln a$ (D) $\frac{a^{\lambda x}}{\lambda \ln a}$
5	$\frac{d}{dx}(\sin \sqrt{x}) =$: (A) $\cos \sqrt{x}$ (B) $\cos \sqrt{x} \cdot \frac{1}{\sqrt{x}}$ (C) $\sqrt{x} \cos \sqrt{x}$ (D) $\cos \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$
6	If $y = x^2 - 1$, then $dy =$: (A) $x dx + c$ (B) $(x-1) dx$ (C) $2x dx + c$ (D) $2x dx$
7	$\int_0^3 \frac{dx}{x^2 + 9} =$: (A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{4}$ (C) 0 (D) $\frac{\pi}{12}$
8	$\int e^x (\sin x + \cos x) dx =$: (A) $e^x \cos x + c$ (B) $e^x \sin x$ (C) $e^x \sin x + c$ (D) $e^x \cos x$
9	$\int \frac{2}{x+2} dx =$: (A) $\ln x+2 + c$ (B) $\ln x+2 ^2 + c$ (C) $\frac{1}{\ln x+2 } + c$ (D) $2 \ln x + c$
10	$\int \frac{1}{\cos^2 x} dx =$: (A) $\frac{1}{\sin^2 x} + c$ (B) $\tan x + c$ (C) $\sec^2 x + c$ (D) $\operatorname{cosec}^2 x + c$

(Turn Over)

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11	The lines represented by $ax^2 + 2hxy + by^2 = 0$ are imaginary if : (A) $h^2 - ab = 0$ (B) $h^2 - ab < 0$ (C) $h^2 - ab > 0$ (D) $h^2 - ab \neq 0$
12	Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel if : (A) $a_1a_2 + b_1b_2 = 0$ (B) $a_1a_2 - b_1b_2 = 0$ (C) $a_1b_2 - a_2b_1 = 0$ (D) $a_1b_2 + a_2b_1 = 0$
13	Inclination of the line joining the points (4, 6) and (4, 8) is : (A) 90° (B) 45° (C) 30° (D) Undefined
14	A region is said to feasible region which is restricted to : (A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant
15	An angle in a semicircle is of measure : (A) 90° (B) 60° (C) 45° (D) 30°
16	The coordinate of the vertices of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is : (A) $(0, \pm b)$ (B) $(\pm b, 0)$ (C) $(0, \pm a)$ (D) $(\pm a, 0)$
17	Focus of the parabola $x^2 - 5y = 0$ is : (A) $(\frac{5}{4}, 0)$ (B) $(0, \frac{5}{4})$ (C) $(0, -\frac{5}{4})$ (D) $(-\frac{5}{4}, 0)$
18	For parabola, value of eccentricity e is : (A) $e = 0$ (B) $e < 1$ (C) $e > 1$ (D) $e = 1$
19	If $\underline{u}, \underline{v}$ and \underline{w} are coterminous edges of a tetrahedron, then its volume is : (A) $[\underline{u} \underline{v} \underline{w}]$ (B) $\frac{1}{3} [\underline{u} \underline{v} \underline{w}]$ (C) $\frac{1}{6} [\underline{u} \underline{v} \underline{w}]$ (D) $\frac{1}{9} [\underline{u} \underline{v} \underline{w}]$
20	A vector perpendicular to both vectors \underline{a} and \underline{b} is : (A) $\underline{a} \cdot \underline{b}$ (B) $\underline{a} \times \underline{b}$ (C) $\frac{\underline{a} \cdot \underline{b}}{ \underline{a} }$ (D) $\underline{b} \cdot \underline{a}$

2. Write short answers to any EIGHT (8) questions :

16

- (i) For $f(x) = \frac{2x+1}{x-1}$, find $f^{-1}(x)$
- (ii) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin \theta}$
- (iii) Discuss the continuity of $f(x)$ at $x = c = 2$, $f(x) = \begin{cases} 2x+5 & \text{if } x \leq 2 \\ 4x+1 & \text{if } x > 2 \end{cases}$
- (iv) Differentiate w.r.t 'x' $(x-5)(3-x)$
- (v) Find $\frac{dy}{dx}$ if $y^2 - xy - x^2 + 4 = 0$
- (vi) Differentiate w.r.t 'θ' $(\sin 2\theta - \cos 3\theta)^2$
- (vii) Find $\frac{dy}{dx}$ if $y = x^2 \ln \frac{1}{x}$
- (viii) Find y_4 if $y = (2x+5)^{3/2}$
- (ix) Apply Maclaurin series expansion to prove that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
- (x) Find extreme values for $f(x) = x^2 - x - 2$
- (xi) Define feasible region.
- (xii) Graph the inequality $x + 2y \leq 6$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Find δy and dy in the case $y = x^2 + 2x$ when x changes from 2 to 1.8
- (ii) Evaluate $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)} dx$, $x > 0$
- (iii) Evaluate $\int a^{x^2} x dx$ ($a > 0, a \neq 1$)
- (iv) Evaluate $\int \sqrt{4-5x^2} dx$
- (v) Evaluate $\int_0^{\frac{\pi}{6}} x \cos x dx$
- (vi) Find area below the curve $y = 3\sqrt{x}$ and above the x-axis between $x=1$ and $x=4$
- (vii) Solve the differential equation $x^2(2y+1)\frac{dy}{dx} - 1 = 0$
- (viii) Find the position vector of the point of division of the line segments joining the following pair of points, in the given ratio, point C with position vector $2\mathbf{i} - 3\mathbf{j}$ and point D with position vector $3\mathbf{i} + 2\mathbf{j}$ in the ratio 4 : 3
- (ix) If $\underline{u} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, $\underline{v} = -\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\underline{w} = \mathbf{i} + 6\mathbf{j} + z\mathbf{k}$ represent the sides of a triangle, find the value of z .

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3. (x) Find the angle between the vectors $\underline{u} = 2\underline{i} - \underline{j} + \underline{k}$ and $\underline{v} = -\underline{i} + \underline{j}$.
- (xi) If $\underline{a} = 4\underline{i} + 3\underline{j} + \underline{k}$ and $\underline{b} = 2\underline{i} - \underline{j} + 2\underline{k}$, find a unit vector perpendicular to both \underline{a} and \underline{b} .
Also find the sine of angle between the vectors \underline{a} and \underline{b} .
- (xii) Find the area of the triangle with vertices A (1, -1, 1), B (2, 1, -1) and C (-1, 1, 2)

4. Write short answers to any NINE (9) questions :

18

- (i) Show that the points A (0, 2), B ($\sqrt{3}$, -1) and C (0, -2) are vertices of a right triangle.
- (ii) Find k so that the line joining A (7, 3), B (k, -6) and line joining C (-4, 5), D (-6, 4) are parallel.
- (iii) Find an equation of line if its slope is 2 and y-intercept is 5.
- (iv) Transform the equation $5x - 12y + 39 = 0$ into two-intercept form.
- (v) Find the distance from the points P (6, -1) to the line $6x - 4y + 9 = 0$
- (vi) Find the point of intersection of lines $3x + y + 12 = 0$ and $x + 2y - 1 = 0$
- (vii) Find the angle between the lines represented by $x^2 - xy - 6y^2 = 0$
- (viii) Find an equation of circle with centre at ($\sqrt{2}$, $-3\sqrt{3}$) and radius $2\sqrt{2}$
- (ix) Find centre and radius of circle $x^2 + y^2 + 12x - 10y = 0$
- (x) Find vertex and directrix of parabola $x^2 = 16y$
- (xi) Find the focus and vertex of parabola $x^2 = 4(y - 1)$
- (xii) Find centre and foci of $4x^2 + 9y^2 = 36$
- (xiii) Find eccentricity and vertices of $\frac{y^2}{16} - \frac{x^2}{9} = 1$

SECTION - II

Note : Attempt any THREE questions.

5. (a) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}$ 5
- (b) If $\frac{y}{x} = \tan^{-1} \frac{x}{y}$ then prove that $\frac{dy}{dx} = \frac{y}{x}$ 5
6. (a) Evaluate $\int \frac{x}{x^4 + 2x^2 + 5} dx$ 5
- (b) Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x-intercept and y-intercept of each is 3. 5
7. (a) Evaluate $\int_0^{\frac{\pi}{4}} (1 + \cos^2 \theta) \tan^2 \theta d\theta$ 5
- (b) Minimize $z = 2x + y$ subject to the constraints
 $x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$ 5
8. (a) If $y = (\cos^{-1} x)^2$, prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$ 5
- (b) Find equations of the tangents to the circle $x^2 + y^2 = 2$ and parallel to the line $x - 2y + 1 = 0$ 5
9. (a) Find volume of the tetrahedron with the vertices (0, 1, 2), (3, 2, 1), (1, 2, 1) and (5, 5, 6) 5
- (b) Find the centre, foci, eccentricity and directrices of ellipse $\frac{(2x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$ 5

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(Academic Sessions 2019 – 2021 to 2021 – 2023)

MATHEMATICS

223-1st Annual-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – II

Maximum Marks : 20

PAPER CODE = 8194 LHR-12-2023

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	The mid point of the line segment joining the foci of an ellipse is called : (A) Vertex (B) Directrix (C) Centre (D) Minor axis
2	If (3 , 5) is mid point of (5 , a) and (b , 7) then : (A) $a=4, b=2$ (B) $a=3, b=3$ (C) $a=7, b=-2$ (D) $a=3, b=1$
3	If 2 and 2 are x and y components of a vector, then its angle with x-axis is : (A) 30° (B) 60° (C) 45° (D) 90°
4	(3 , 2) is not a solution of the inequality : (A) $x-y>1$ (B) $x+y>2$ (C) $3x+5y>8$ (D) $3x-7y<3$
5	$\underline{i} \times \underline{j} = :$ (A) \underline{k} (B) \underline{i} (C) $-\underline{k}$ (D) \underline{j}
6	Slope of line $3x-2y+5=0$ is : (A) $-\frac{2}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$
7	Length of the diameter of the circle $(x+5)^2+(y-8)^2=12$: (A) $2\sqrt{3}$ (B) 12 (C) 24 (D) $4\sqrt{3}$
8	Transverse axis of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is : (A) $x=\frac{a}{e}$ (B) $y=0$ (C) $x=0$ (D) $y=\frac{a}{e}$
9	Equation of line in slope intercept form is : (A) $y=mx+c$ (B) $\frac{x}{a}+\frac{y}{b}=1$ (C) $y-y_1=m(x-x_1)$ (D) $x\cos\alpha+y\sin\alpha=p$
10	The condition for a line $y=mx+c$ to be tangent to the circle $x^2+y^2=a^2$ is that : (A) $c=\pm m\sqrt{1+a^2}$ (B) $c=\pm a\sqrt{1+m^2}$ (C) $c=\pm a\sqrt{1-m^2}$ (D) $c=\pm\sqrt{1-m^2}$

(Turn Over)

(2)

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11	$f(x) = f(o) + xf'(o) + \frac{x^2}{2!} f''(o) + \frac{x^3}{3!} f'''(o) + \dots$ is called : (A) Taylor's series (B) Binomial series (C) Maclaurin's series (D) Laurent series
12	$\lim_{x \rightarrow 0} \frac{\sin 7x}{x} = :$ (A) 7 (B) -7 (C) $-\frac{1}{7}$ (D) $\frac{1}{7}$
13	$\int \frac{e^x}{e^x + 1} dx = :$ (A) $\ln(e^x + 1) + c$ (B) $\ln e^x + c$ (C) $e^{-x} + c$ (D) $e^x + c$
14	$\frac{d}{dx} (x^2 + 1)^2 = :$ (A) $2x(x^2 + 1)$ (B) $\frac{(x^2 + 1)^3}{3}$ (C) $2(x^2 + 1)$ (D) $4x(x^2 + 1)$
15	$\int \sin^2 x dx = :$ (A) $\frac{x}{2} - \frac{\sin 2x}{4} + c$ (B) $\frac{x}{2} + \frac{\sin 2x}{4} + c$ (C) $\frac{x}{2} - \frac{\sin 2x}{2} + c$ (D) $\frac{x}{2} + \frac{\sin 2x}{2} + c$
16	$\frac{d}{dx} (\tan x^2) = :$ (A) $\sec^2 x^2$ (B) $2x \sec^2 x^2$ (C) $-\sec^2 x^2$ (D) $-2x \sec x^2$
17	$\int e^x (\cos x - \sin x) dx = :$ (A) $e^x \sin x + c$ (B) $e^x \cos x + c$ (C) $e^x \tan x + c$ (D) $e^x \cot x + c$
18	If $f(x) = \sin x + \cos x$ then $f(x)$ is : (A) Even function (B) Odd function (C) Neither even nor odd (D) Constant function
19	$\int_0^3 \frac{1}{9 + x^2} dx = :$ (A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{12}$ (C) $\frac{-\pi}{4}$ (D) $\frac{\pi}{12}$
20	$\frac{d}{dx} (f(x) \sin x) = :$ (A) $f(x) \cos x + f'(x) \sin x$ (B) $f'(x) \sin x - f(x) \cos x$ (C) $f'(x) \cos x$ (D) $f'(x) \cos x + f(x) \sin x$

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(Academic Sessions 2019 – 2021 to 2021 – 2023)

MATHEMATICS

223-1st Annual-(INTER PART – II)

Time Allowed : 2.30 hours

PAPER – II (Essay Type)

GROUP – II

Maximum Marks : 80

SECTION – I

C HR-12-2-23

2. Write short answers to any EIGHT (8) questions :

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- (i) Find the domain and range of $f(x) = \sqrt{x^2 - 4}$
- (ii) Show that $x = a \sec \theta$, $y = b \tan \theta$ represents the equation of hyperbola.
- (iii) If $f(x) = -2x + 8$, find $f^{-1}(x)$ and $f^{-1}(-1)$
- (iv) Differentiate $(3 - x)(x - 5)$ w.r.t 'x'
- (v) Find derivative of $\sqrt{\frac{1+x}{1-x}}$
- (vi) If $y = x^4 + 2x^2 + 2$, prove $\frac{dy}{dx} = 4x\sqrt{y-1}$
- (vii) Find the derivative of $(x^3 + 1)^9$ w.r.t. 'x'
- (viii) Find $\frac{dy}{dx}$ if $y^3 - 2xy^2 + x^2y + 3x = 0$
- (ix) Differentiate w.r.t. variable involved of $\tan^3 \theta \sec^2 \theta$
- (x) Find $\frac{dy}{dx}$ if $y = a^x$
- (xi) Define feasible region.
- (xii) Graph the feasible region $2x - 3y \leq 6$, $x \geq 0$, $y \geq 0$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Using differentials to find $\frac{dy}{dx}$ if $xy - \ln x = c$
- (ii) Evaluate $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$
- (iii) Evaluate $\int \frac{e^x}{e^x + 3} dx$
- (iv) Evaluate $\int \ln x dx$
- (v) Evaluate $\int_{-6}^2 \sqrt{3-x} dx$
- (vi) Find the area bounded by cos function from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
- (vii) Solve the differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$
- (viii) Find the magnitude of the vector $\underline{u} = \hat{i} + \hat{j}$
- (ix) Find direction cosines of $\vec{v} = 3\hat{i} - \hat{j} + 2\hat{k}$
- (x) Calculate the projection of \vec{b} along \vec{a} if $\vec{a} = \hat{i} - \hat{k}$; $\vec{b} = \hat{j} + \hat{k}$
- (xi) If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$; $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, find $\vec{b} \times \vec{a}$
- (xii) Prove that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.

(Turn Over)

4. Write short answers to any NINE (9) questions :

- (i) Find the equation of the straight line whose slope is 2 and y-intercept is 5.
- (ii) Using slopes, show that the triangle with its vertices A (6 , 1) , B (2 , 7) and C (- 6 , - 7) is a right triangle.
- (iii) Find an equation of the line through (- 4 , 7) and parallel to the line $2x - 7y + 4 = 0$
- (iv) Find h such that A (- 1 , h) , B (3 , 2) and C (7 , 3) are collinear.
- (v) Write intercepts form of equation of straight line.
- (vi) Check whether the following lines are concurrent or not
 $3x - 4y - 3 = 0$
 $5x + 12y + 1 = 0$
 $32x + 4y - 17 = 0$
- (vii) Find the slope and inclination of the line joining points (- 2 , 4) and (5 , 11)
- (viii) Find an equation of circle with centre at $(\sqrt{2} , -3\sqrt{3})$ and radius $2\sqrt{2}$
- (ix) Define focus and directrix of the parabola.
- (x) Find the centre and foci of the ellipse $x^2 + 4y^2 = 16$
- (xi) Find equation of tangent to $y^2 = 4ax$ at (x_1, y_1)
- (xii) Show that the equation $5x^2 + 5y^2 + 24x + 36y + 10 = 0$ represents a circle. Find its centre.
- (xiii) Find an equation of the ellipse with given data : Foci (0 , - 1) and (0 , - 5) and major axis of length 6.

SECTION - II

Note : Attempt any THREE questions.

5. (a) If θ is measured in radians then prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ 5
- (b) Find $\frac{dy}{dx}$ if $y = (1 + 2\sqrt{x})^3 \cdot x^{\frac{3}{2}}$ 5
6. (a) Evaluate $\int \ln(x + \sqrt{x^2 + 1}) dx$ 5
- (b) Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x-intercept and y-intercept of each is 3. 5
7. (a) Solve the differential equation $2e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$ 5
- (b) Maximize $f(x, y) = x + 3y$ subject to constraints
 $2x + 5y \leq 30$, $5x + 4y \leq 20$, $x \geq 0$, $y \geq 0$ 5
8. (a) If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ 5
- (b) Find equations of tangents to the circle $x^2 + y^2 = 2$ perpendicular to the line $3x + 2y = 6$ 5
9. (a) Show that the equation $9x^2 - 18x + 4y^2 + 8y - 23 = 0$ represents an ellipse. Find its elements and sketch its graph. 5
- (b) Prove that in any triangle ABC $c = a \cos B + b \cos A$ 5